Physics **Review of Fundamental Mathematics, Measurement & Graphing Techniques**

Introduction: One of the coolest concepts concerning science is that all of it is tied together by a few fundamental mathematical relationships. The science that examines these relationships is called physics. Some people find physics difficult because it requires constant use of various branches of mathematics. But the mathematics needed is a kind seldom emphasized in math courses, and rightly so. Mathematicians are concerned with rigorous proofs and generalized cases. Physicists and engineers are more concerned with how to use math as a tool to describe phenomena. Instead of rigorous proofs, we want plausibility arguments. Instead of generalized cases, we want specific applications - usually with numbers attached. In a math course, for example, the area of a circle with a radius of 2 is equal to $\pi r^2 = 4\pi$. For our purposes, such a statement is meaningless, and thus useless. The radius must have units attached: 2 meters. Since the value of the radius must have been determined by a measurement, we must specify the precision of that measurement in terms of the significant digits of the number. Is the radius 2 meters or 2.00 meters? Finally, for our purposes, the area of the circle is about 12 meters^2, not about 4π meters^2.

The goal of this unit is to review some basic math, start using math in scientific application, and lay the groundwork for learning some problem solving techniques to be used throughout the rest of the course.

Performance Objectives: Upon completion of the exercises and problems in this unit and when asked to respond either orally or on a written test, you will:

- ✓ Use significant digits as a means of stating the precision of measurements.
- ✓ Perform calculations using significant digits.
- ✓ List the standard metric units for mass, length, and time.
- ✓ Interpret metric prefixes.
- ✓ Perform metric conversions.
- ✓ Use scientific notation for large and small numbers.
- ✓ Add, Subtract, Multiply, and Divide numbers in scientific notation.
- ✓ State the fundamental units of measurement.
- ✓ Distinguish between fundamental and derived units.
- ✓ Transpose simple equations simply and efficiently.
- ✓ Evaluate the validity of an equation by dimensional analysis.
- ✓ Distinguish between dependent and independent variables.
- ✓ Interpret the meaning of straight line, hyperbola, and parabola when plotting graphs.
- ✓ Recognize linear and direct variations.
- ✓ Recognize inverse variations.
- ✓ Evaluate graphic representation of experimental data.
- ✓ Use right triangle trigonometry.
- ✓ Use the law of cosines.
- ✓ Use the law of sines.

"I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; But when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; It may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the state of Science, whatever the matter may be." *Lord Kelvin (1824 – 1907)*

Law of Significant Digits: What are significant digits? When you make a measurement, significant digits are all digits that are known with certainty (can be read from the measuring instrument) and **ONE** uncertain or estimated digit that best represents the value of measurement. The number of significant digits in a measurement depends on the instrument used to measure.

When you read a measurement written by someone else, the number of significant digits tells you how good the measuring instrument is for the quantity being measured. We have adopted some rules to determine the number of significant digits:

- ✓ Digits other than zero (1 – 9) are **ALWAYS** significant! (Pretty straight forward – right?)
- ✓ Only two occasions exists where a zero would be considered significant…
  - ➢ Zero(s) between other significant digits ARE significant!
  - ➢ End zero(s) to the right of the decimal ARE significant!

  (If you want to show that other end zeros are significant, you must use scientific notation.)

Mathematical Operations with Significant Digits: If you perform mathematical operations with significant digits, you must remember that your answer can have only one uncertain place.

- ✓ In addition or subtraction, the result of the operation should contain only the number of decimal places of the quantity with the fewest number of decimal places (least precise).
- ✓ In multiplication or division, the result should contain no more significant digits than are contained in the measurement with the least number of significant digits.

How many significant digits are in each of the following:

1.) 15 m
2.) 37.5 m
3.) 100 cm
4.) 1000.00 cm
5.) 1000014 mm
6.) 10250 mm
7.) 40 Hm
8.) 42.500 Hm
9.) 10000 nm
10.) 17500.00 nm
11.) 4.20×10^6 μm
12.) 3×10^8 pm

Write the following measurements in scientific notation:

13.) 200 cm
14.) 0.003 m
15.) 35,700 nm
16.) 890,200 nm
17.) 63,000 mm
18.) 0.556 km
19.) 0.07007 kg
20.) 0.0000006 Mg

Express in ordinary notation:

21.) 9.81×10^8 m/s
22.) 3.11×10^{-4} km
23.) 4.27×10^3 mg
24.) 6.70×10^{-2} kg

Perform the indicated operations:

25.) (4.2×10^4) g + (3.6×10^4) g
26.) (1.66×10^{-19}) Em + (2.30×10^{-20}) Em
27.) (6.0×10^{-3}) kg – (2.0×10^{-4}) kg
28.) (2.2×10^{12}) fm – (8.0×10^{11}) fm
29.) (2.5×10^7) mm x (2.5×10^{16}) mm
30.) (9.5×10^{11}) cm x (6.0×10^{-8}) cm
31.) (1.96×10^4) m^2 / (1.4×10^{-3}) m
32.) (6.25×10^{-3}) m^3 / (2.4×10^4) m^2

Algebra Review:

Solve each of the following equations for v.

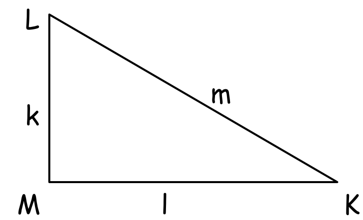
33.) $s = vt$
34.) $E = mv^2/2$
35.) $a = v^2/2s$
36.) $F = mv^2/r$
37.) $v/a = b/c$

Solve for the indicated quantity:

38.) Solve for R: $F = kq_1q_2/R^2$
39) Solve for t: $s = gt^2/2$
40.) Solve for m: $E = mv^2/2$
41.) Solve for a: $F = ma$
42.) Solve for m: $(mv^2)/2m = p^2/2m$
43.) Solve for t: $x = vt + at^2/2$

Trigonometry Review:

44.) a.) Which side of the triangle is opposite angle K? b.) Which side is adjacent to angle L? c.) Write equations for the sine, cosine, and tangent of angle K.



45.) Find the size of the angles associated with each trigonometric function below.

a.) sin A = 0.500 d.) sin A = 0.985 g.) cos A = 0.707
b.) sin A = 0.707 e.) tan A = 1.00 h.) cos A = 0.866
c.) tan A = 2.050 f.) tan A = 0.364

46.) One angle of a right triangle is 20.0°. The length of the hypotenuse is 6.00 cm. A.) Draw the triangle to scale and measure the lengths of the other two sides. B.) Use trigonometry to calculate the lengths of these two sides.

Unit Conversion Review: In 1971, the 14th General Conference on Weights and Measures picked seven quantities as fundamental or base quantities, thereby forming the basis of the International System of Units (Systeme Internationale) abbreviated SI and commonly called the metric system. We will still use the MKS units to measure the three fundamental quantities of length, mass, and time. Many other SI quantities are derived by multiplying or dividing fundamental units. Examples of derived units would be m^2 to measure area, and km/s to measure speed.

The metric system allows us to conveniently express very large or very small numbers by attaching prefixes to base units. For example, 5000000000 watts can be written as 5.00×10^9 watts or 5.00 gigawatts.

We can convert from one unit to another by using conversion factors (a ratio of units that is equal to 1). Recall that in a conversion factor the units obey the same rules as do algebraic variables and numbers.

Factor	Prefix	Symbol
10^{24}	yotta-	Y
10^{21}	zetta-	Z
10^{18}	exa-	E
10^{15}	peta-	P
10^{12}	tera-	T
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{2}	hecto-	h
10^{1}	deka-	da

Factor	Prefix	Symbol
10^{-24}	yocto-	y
10^{-21}	zepto-	z
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{-1}	deci-	d

Metric-to-Metric Conversions:

47.) 756 mm to m *0.756 m*
48.) 2.0 km to microns *2.0×10^9 microns*
49.) 1 km/yr to m/s *3×10^{-5} m/s*
50.) 10 cm^2 to mm^2 *1000 mm^2*
51.) 5.2×10^{15} pm to Gm *5.2×10^{-6} Gm*

The United States is the only major country (probably the only country) that has not officially adopted the International System of Units. Here are some conversion factors we will be practicing with and using:

1 inch = 2.54 cm 1 mile = 1.6 km 1 lb = 4.45 N
1 lightyear = 9.46×10^{12} km 1 furlong = 220 yards 1 fortnight = 2 weeks

English-to-Metric Conversions:

52.) 245 cm to inches *96.5 inches*
53.) 7.3 miles to km *12 km*
54.) 1.0 furlongs/fortnight to lightyears/exasecond *1.8×10^{-2} Ltyrs / Es*

Prefix Phun:

55.) 10^6 phones
56.) 10^{-6} phones
57.) 10^1 cards
58.) 10^{12} bull
59.) 10^{-1} mates
60.) 10^{-2} pedes
61.) 2×10^2 withits
62.) 2×10^3 mockingbird

Unit Analysis Review: Dimensional Analysis!

Perform unit / dimensional analysis on the following. Indicate possible solutions for the question marks.

63.) ? = m/V where: m is measured in kg
V is measured in cm^3

64.) $? = v_0\Delta t + 1/2a\Delta t^2$ where: v_0 is measured in m/s
t is measured in s
a is measured in m/s^2

65.) $?^2 = v_0^2 + 2a\Delta x$ where: Δx is measured in m

66.) $? = Gm_1m_2/d^2$ where: G is measured in N m^2/kg^2
m is measured in kg
d is measured in m

67.) ? = ma

68.) ? = mg where: g is gravity's acceleration

Rules for Graphing Experimental Data: The relationship between two physical quantities is often obtained in the form of a table of data. At other times, the relationship is expressed by a mathematical equation. In either case, it may be easier and more useful to visualize the relationship by means of a graph.

Suppose you carry out an investigation that allows you to measure the speed of a ball as it rolls down a ramp. You can measure the speed at certain time periods. Essentially you are controlling (manipulating) the length of time the ball is allowed to roll. The speed of the ball depends on how long it rolls. The variable you control is the independent variable. The one you measure is the dependent variable.

RULE 1: Decide which variable is independent and which is dependent. In this case, time is considered to be the independent variable because the distance traveled depends on how long the ball has been rolling. The independent variable is the one manipulated in a scientific experiment. It is to be graphed on the horizontal axis. The dependent variable (velocity in this case) is to be plotted on the vertical axis.

RULE 2: Choose (or even calculate) scales that will stretch the variables across the entire page, using as much of the graph paper as possible. However, when you do this, choose a scale that makes it easy to locate points. Never let five squares equal six units, for example. A scale having five squares equal to ten units would be better. It is not necessary to use the same scale in both directions, even when both variables are expressed in the same units.

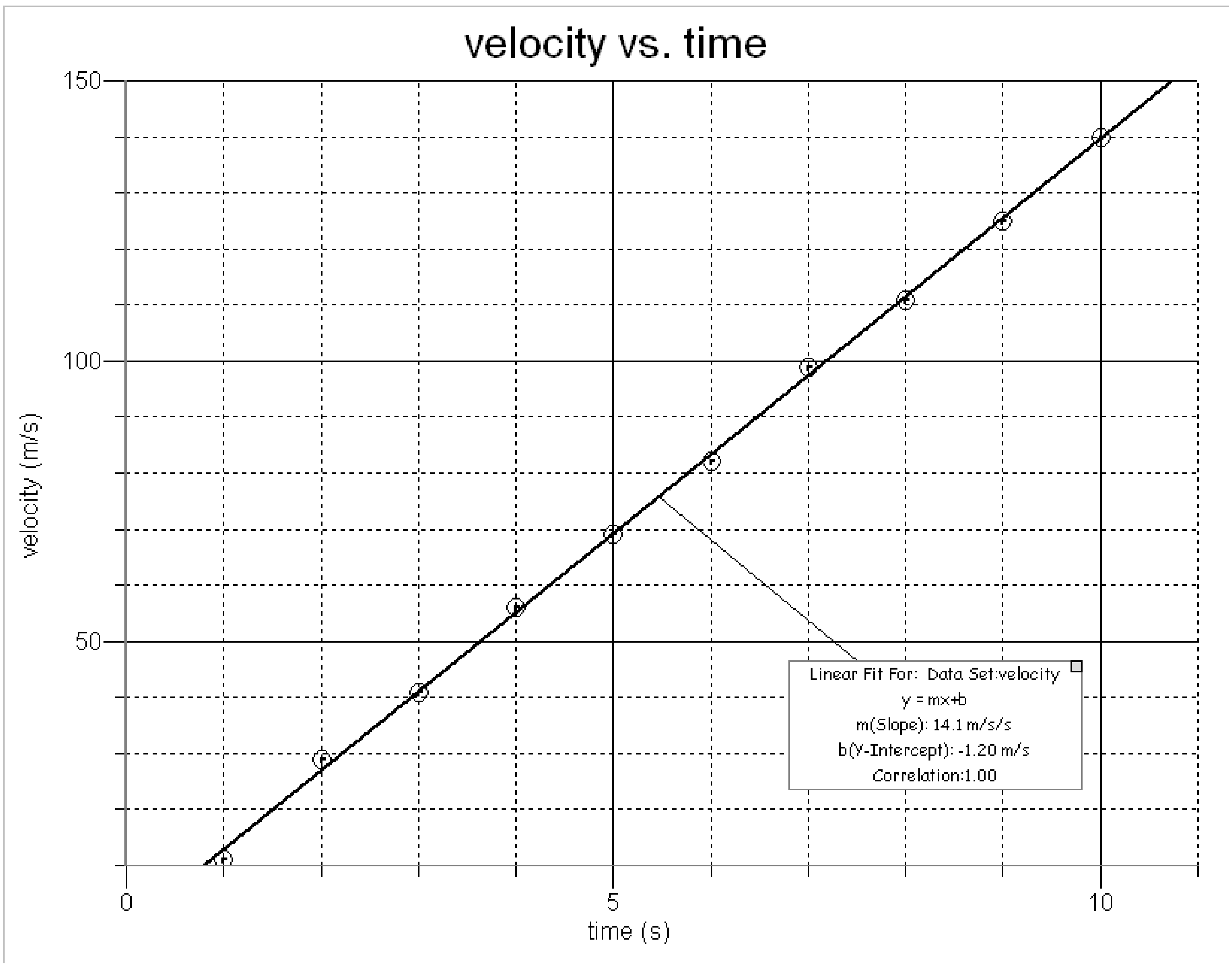
RULE 3: Put numbers on the axes according to your scale. Label each axis to indicate the quantity that is being graphed as well as the unit that the quantity was measured in. Place these labels along the sides of your graph.

RULE 4: Locate each data point by making a small dot in pencil. When you are sure that you have made no mistake in locating points, ink each dot and draw a circle around it in ink.

RULE 5: With a well sharpened pencil, lightly draw the smooth line that you think fits the data points best – never connect the dots. If the points seem to lie along a straight line, draw the line that best fits with a ruler, preferably a transparent one, so that you can see all of the points while you select the position of the line. Try to get as many points above the line as below the line. If the points seem to lie along a curve, draw a smooth curve. Do not try to force your line to go through all points!

Experimental error will usually cause some of the points to be off the line. For an experiment done carefully, with very accurate equipment, the points may be very close to the line. Because you have located the data points in ink and your line in pencil, you will find it easy to change the line of you are not satisfied with it.

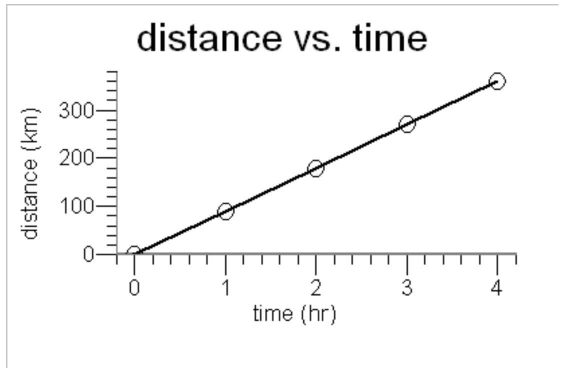
RULE 6: Give your graph a title – "Dependent vs. Independent." See the sample graph below!



Now-a-days, we have computers to take a lot of the tediousness away from graphing. However, these basic rules still apply to all graphs and should be considered when using software to produce graphs of data that you have so carefully acquired in the lab.

Graphic Interpretation of Data: When analyzing laboratory data, it is often useful to determine if there is some type of relationship among the variables in the data. One method of testing this possibility is to graph the data and to look for some type of proportionality that may exist. For example, consider the following data collected during a car trip.

time (hr)	distance (km)
0	0
1	90
2	180
3	270
4	360

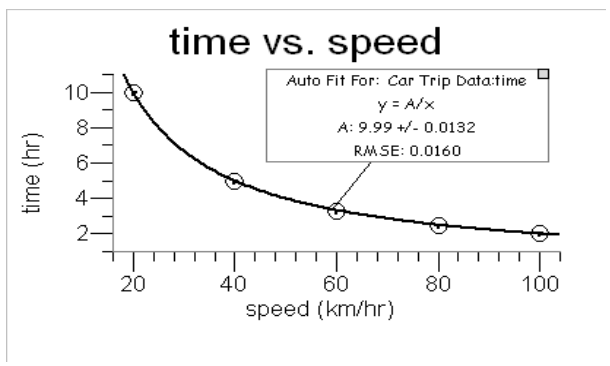


A distance vs. time graph of this data produces a graph that looks like the above. The graph is "linear" indicating that "distance varies directly with time," or that "distance is directly proportional to time." Sometimes we symbolize this relationship as $d \propto t$ or $d = kt$, where d is distance, t is time and k is the proportionality constant or slope of the line.

If the y-intercept is not zero, the general form of the equation would be $y = mx + b$, where m is the slope of the line and b is the y-intercept. There is a general linear dependence, but it is not directly proportional. Linear or proportional dependence in opposite directions corresponds to a straight line graph with a negative slope. An example of this would be a graph of speed vs. time for a car slowing down.

As another example, consider the data collected from several trips in which the driver drove the same distance during each trip, but at a different speed during each trip. The graph of time versus speed has quite a different shape from the previous graph.

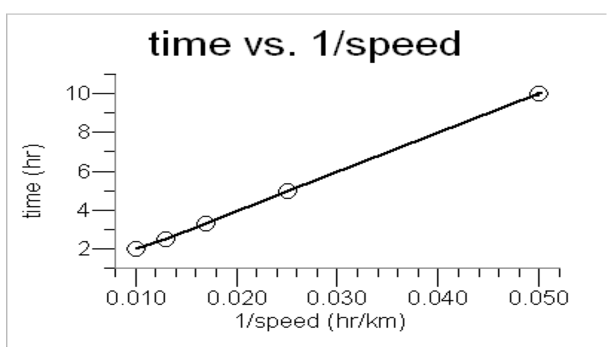
speed (km/hr)	time (hr)
20	10.0
40	5.0
60	3.3
80	2.5
100	2.0



In this case, as speed increases – time decreases, and vice versa. It appears that some type of inverse relationship exists between time and speed – that "time varies inversely with speed" or time is inversely proportional to speed. We can write this simply as $t \propto 1/v$ or as $t = k/v$.

Sometimes it is useful to "manipulate or linearize" data in order to examine a suspected relationship more closely. Using the data from above, we can look at the "inverse" type of relationship and attempt to plot time as a function of the inverse of speed. If this is a simple inverse relationship, such a plot will yield a straight line (resulting in a linear or direct relationship).

speed (km/hr)	time (hr)	1/speed $(km/hr)^{-1}$
20	10.0	0.050
40	5.0	0.025
60	3.3	0.017
80	2.5	0.013
100	2.0	0.010

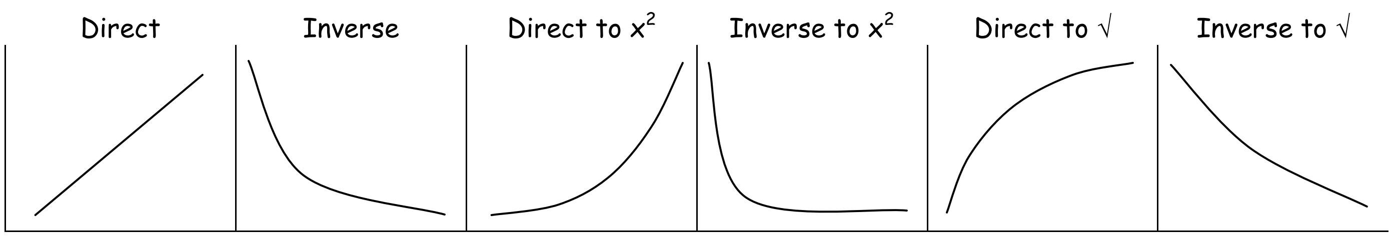


The straight line of this last plot tells us that "time varies directly with the inverse of speed," or that "time is inversely proportional to speed." The equation is $t = k(1/v)$ where t is time, v is speed and k is the proportionality constant or the slope of the line.

Relationship Statements: Listed below are relationships frequently encountered in beginning physics. For each relationship, there is in example of a verbal description of the relationship, a proportionality statement, and an equality statement.

- ✓ **Direct:** The distance driven in a car varies directly with the time of travel (at constant speed). The proportionality statement would be written as: $d \propto t$. The equality statement would be written as: $d = kt$.
- ✓ **Inverse:** The time needed to drive a fixed distance in a car varies inversely with the driving speed. The proportionality statement would be written as: $t \propto 1/v$ or $t \propto v^{-1}$. The equality statement would be written as: $t = k/v$ or $vt = k$ or $t = kv^{-1}$.
- ✓ **Direct to the Square:** The area of a circle varies directly with the square of the circle's diameter. The proportionality statement would be written as: $A \propto d^2$. The equality statement would be written as: $A = kd^2$. What is k equal to in this case?
- ✓ **Inverse to the Square:** The radiant heat received from a bonfire varies inversely with the square of the distance from the fire. The proportionality statement would be written as: $H \propto 1/d^2$ or $H \propto d^{-2}$. The equality statement would be written as: $H = k/d^2$ or $H = kd^{-2}$.
- ✓ **Direct to the Square Root:** The period of a simple pendulum varies directly with the square root of the length of the pendulum. The proportionality statement would be written as: $T \propto \sqrt{L}$ or $T \propto L^{1/2}$. The equality statement would be written as: $T = k\sqrt{L}$ or $T = k(L)^{1/2}$.
- ✓ **Inverse to the Square Root:** The frequency of a vibrating string varies inversely with the square root of the linear density of the string. The proportionality statement would be written as: $f \propto 1/\sqrt{\mu}$ or $f \propto \mu^{-1/2}$. The equality statement would be written as: $f = k/\sqrt{\mu}$ or $f = k(\mu)^{-1/2}$.

Basic Shapes of Relationship Graphs:



There are only subtle differences (in the amount of "curve") between the graphs of an inverse relationship, an inverse to a square relationship, and an inverse to a spare root relationship. Therefore, until you become more familiar with many different types of physical phenomena which may exhibit these different types of relationships, it is best not to make assumptions about the relationships between the variables until you have investigated as much as possible!