AP Physics-B

Simple Harmonic Motion

Introduction: The main objective of the previous units was to discover that motion of a body can be predicted if the initial conditions describing its state of motion and the external forces acting it are known. If the force varies in time, the velocity and acceleration of the body will also change with time. In previous units, we studied circular motion - some of the problems and labs explored, involved objects that traveled in the same circular path more than once. Circular motion is an example of periodic motion. A swinging pendulum and a vibrating guitar string are examples of vibrational or oscillatory motion. In this type of motion, an object moves back and forth over the same path. Oscillatory motion is another example of Simple Harmonic Motion or Periodic Motion.

Understanding the simple motion of a mass on a spring will give you insight into many other systems. The number of systems that exhibit oscillatory motion is extensive. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar waves, and radio waves are characterized by oscillating electric and magnetic field vectors; and in alternating-current circuits, voltage, current, and electrical charge vary periodically with time. Simple Harmonic Motion (SHM) is the name given to the oscillatory motion that occurs whenever the restoring force on a body is proportional to the displacement of the body from its equilibrium position. For a mass on a spring, the restoring force is described by F = -kx, known as Hooke's Law. The restoring force is also proportional to the acceleration. We have SHM whenever the acceleration is proportional to the displacement. Either of these conditions defines SHM. The Period (T) of the SHM is the time for one complete vibration. The frequency (f) is given by f = 1/T.

The projection of uniform circular motion upon one axis also describes SHM. The radius of the reference circle equals the amplitude (A) of the SHM. Amplitude is the maximum displacement from the equilibrium position. The speed (v) of a reference particle on the circumference of the circle equals the maximum velocity of a particle in SHM as it passes through the center (equilibrium position); $v = 2\pi (A/T)$ (Conservation of Energy Considerations)

The period of a mass on a vibrating spring is given by $T = 2\pi (m/k)^{1/2}$ where m is the mass of the object attached to the spring and k is the force constant of the spring. This is the same k an in the Hooke's Law formula, F = -kx, which we studied earlier. The units for k are N/m or the amount of force per amount of compression or stretch. If the force constant is exceeded, permanent distortion occurs. In systems where the only force acting on the system is the restoring force, no loss in mechanical energy occurs. In mechanical systems, retarding (or frictional) forces are always present. Such forces reduce the mechanical energy of the system as motion progresses, and the oscillations are said to be damped. If an external driving force is applied such that the energy loss is balanced by the energy input, we call the motion a forced oscillation.

Performance Objectives: Upon completion of this unit and when asked to respond either orally or on a written test, you will:

- Distinguish between simple harmonic motion and other types of regular motion. Recognize simple harmonic motion as the projection of circular motion on one axis.
- Be able to state the definitions for period, frequency, and angular frequency. Show how these quantities are related to each other. Show that the period depends on the inertial and elastic properties of the system undergoing SHM.
- State the definition for amplitude. Show that the period is independent of the amplitude.
- State the energy relations in SHM. Solve problems involving energy in SHM.
- Recognize an equation for position as a function of time. Using the position function, identify the amplitude, angular frequency and phase angle. Determine the period, mass, or spring constant from the position function.
- Solve problems involving masses on a spring, other vibrational motion, and the simple pendulum.

"Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it."-Galileo Galilei

Textbook Reference: Physics (Sixth Edition) - Chapter 13

Exercises:

1.) The piston of an engine moves a total of 0.2 m from one extreme point to the other. a.) What is the amplitude? 0.1 m
b.) What is the displacement when the piston is 0.1 m from one end of its stroke? zero

2.) A vibrating reed in a harmonica makes 200 vib/s. What is the period? 0.005 s

3.) What is the time required for 2400 vibrations of a loaded spring if the frequency is 4 vib/s? 600 s

4.) Compute the time for one vibration of a spring whose force constant is 16 N/m, if the load has a mass of 1.0 kg. 1.57 s

5.) An object that weighs 25 N is moving with SHM of frequency 60.0 vib/min and amplitude 1.5 m. What is the force constant? What is the restoring force when the object is at one of the end points of its motion? 101 N/m - 152 N

6.) A 30.0 kg load hangs from a long light spring. when pulled down 20.0 cm below its equilibrium position and released, it vibrates with a period of 2.0 s. a.) What is its velocity as it passes through the equilibrium position? 62.8 cm/s
b.) By how much will the spring shorten if the load is removed? 0.99 m

7.) A raft weighing 4000.0 N is floating in a pond. When a 1000.0 N man climbs on board, the raft sinks 0.04 m deeper into the water to a new equilibrium position. If the man jumps off, how many vertical vibrations does the empty raft make in 5.0 seconds? (Hint: First calculate the force constant, then calculate the period of vibration.) $k = 2.5 \times 10^4 N/m$ 6.3 vib

8.) The springs of a 1000.0 kg car compress vertically 0.500 cm when a 100.0 kg man steps in. With the man in the car, how many vibrations per second does the car make after getting jarred while going over a speed bump? (Hint: Find the spring constant for the compression caused by the man getting into the car.) 2.13 vib/s

9.) The prong of a tuning fork vibrates in SHM, with a period of 1/500 seconds. The prong moves through 2.00 mm from one amplitude to the other. What is its maximum speed? 3.14 m/s

Simple Pendulum: As a special case, the motion of a simple pendulum turns out to be approximately SHM if the amplitude of swing is not too great. The period of a simple pendulum is given by $T = 2\pi (I/g)^{1/2}$, where I is the length of the pendulum and g is the acceleration due to gravity at the location of the pendulum. This equation can be used to measure the acceleration due to gravity. Notice that the period of a pendulum does not depend on the mass of the body or on the amplitude of the swing. In any location, the period depends only on the length of the pendulum (I) since g is constant for a given location.

10.) What would be the period of a pendulum suspended from the top of a tall building on a light string 353 m long? 37.7 s

11.) Compute the length of a grandfather clock pendulum that ticks once each second. (Hint: It ticks twice during each complete vibration.) 0.994 m

12.) What is the acceleration due to gravity on a planet where a space explorer's simple pendulum 0.400 m long vibrates 100 times in 240 seconds? 2.74 m/s^2

13.) A pendulum 0.25 m long has a period of 1.1 seconds. What is the period of a pendulum in the same location if it has a length of 0.10 m? 0.70 s

14.) a.) If a pendulum clock loses time, should the pendulum be made shorter or longer? b.) Explain!

15.) A small pendulum is mounted in a space vehicle to measure acceleration. Before launch, the pendulum vibrates at the rate of 150 vib/s. At one point during the launch, the vibration rate is 320 vib/s. What is the acceleration of the vehicle at that point? 45 m/s^2

16.) Derive a formula for the average and for maximum acceleration of a particle in SHM in terms of the amplitude (A) and the period (T). Check you answer for dimensional consistency. $a_{avg} = 8\pi A/T^2$