AP Physics-B **Electric Fields**

Introduction: In much the same way that the Earth exerts a gravitational force on us as we exist in its gravitational field, a charged object exerts an electric force on charges that exists within its electrical field. Likewise, recalling that the Earth's gravitational field weakens as a person gets farther and farther away from the center of the earth so too does the electrical field around a charged object (and consequently the electrical force on nearby charges) decrease as the distance away from the charged object grows. The concept of an electrical field around a charge can be thought of in terms of the force per unit charge that a charged object would apply to test charges brought within its field.

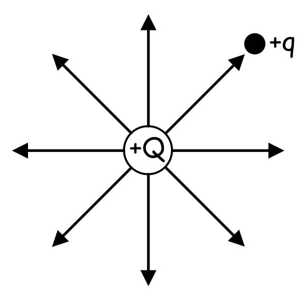
Performance Objectives: Upon completion of the readings, exercises, and problems and when asked to diagram, demonstrate, or respond either orally or on a written test, you will:

- Explain the concept of an electric field. State the units used to measure electric fields. Solve problems involving the strength of electric field on charges at specific locations within the field.
- Recognize electric field lines as a picture of the electric field around a charged object.
- Define potential difference.
- Understand that the work done against an electric field is equivalent to the potential difference between two points. Solve problems involving electrical potential difference.
- Understand that a capacitor can store charge. Define the unit of capacitance.
- State the units used to measure electric fields.

Textbook Reference: Physics: Wilson, Buffa, Lou: Chapter 15

The Electric Field:

According to Coulomb's Law, a charged sphere (of charge +Q) will repel a small test charge (of charge +q) that is brought nearby. This test charge would be repelled radially outward from the charged sphere. This repulsive force will accelerate the test charge according to Newton's Second Law ($F_{net} = ma$). Therefore, +Q is said to set up an electric field that produces a force on any charge that enters this field! This test charge would experience a stronger force as work was done on it to bring it closer to the charged sphere.



Electric fields can be "mapped" around charges using electric field lines (as shown in the diagram above). These lines give a visual representation of the field surrounding a charge and may be used to predict the behavior of test charges brought within this field. **Concerning Electric Field Lines:**

- ✓ EF Lines begin on positive charges and end on negative charges (meaning they have a beginning and an end!)
- ✓ EF Lines are perpendicular to the surface.
- ✓ EF Lines never cross one another.
- ✓ The density of EF Lines is related to the strength of the electric field.

Recall from Newton's Universal Law of Gravitation that $g = G \cdot m_e / R_e^2$ which has units of Force per unit mass (N/kg). This constant is essentially the gravitation field strength constant that we've used so often in this course. In a similar vein, the electric field strength may be expressed as:

$E = F / q$ which has units of Force per unit charge or N/C

Electric Potential:

Because this E-field from the charged sphere is directed radially outward and the strength of the field increases as the distance from a charged sphere gets small, we would have to do an increasing amount of work in order to bring test charges in toward the sphere. In the same way that when we did work on an object to lift it in a gravitational field thus changing the object's gravitational potential energy, electrical potential energy of a charge may be changed by doing work on it in an electrical field.

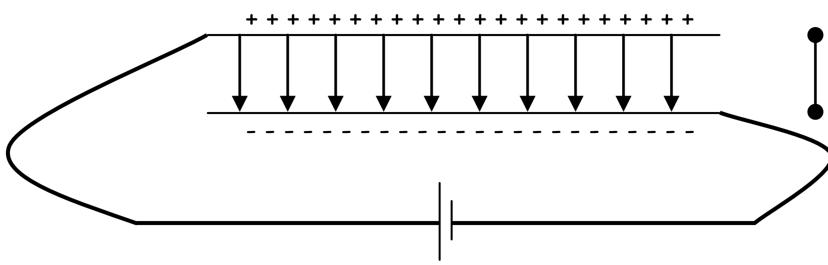
Remember that work done in the conservative gravitational field has been defined as $W = -F \cdot \Delta x = -\Delta U_G$. Since both electric and magnetic fields are also conservative fields, we can define the work done in an electrical field as:

$$W = -F_E \cdot \Delta r = -(k \cdot q_1 \cdot q_2 / r^2) \cdot \Delta r = -k \cdot q_1 \cdot q_2 / r = -\Delta U_E$$

Since the electrical potential energy at any location of the same radius from our test sphere would have the same value, when doing work in moving a test charge closer to the charged sphere, we will be changing the electrical potential of our test charge. This potential difference is also called voltage ($V = U_E / q$) and exists whether the test charge is there or not. Therefore:

$$U_E = q \cdot V = k \cdot q_1 \cdot q_2 / r$$

Calculating the work can be difficult in the case of a point charge since the electric field can vary. Therefore, it would be beneficial to consider a uniform electric field which could be achieved by separating two charged plates (which can store electric charge) by a small distance (d).



Between the two plates, we have a uniform electric field given by:

$$E = V / d$$

Now that we have a uniform electrical field, let's look at the similarities between "G" fields and "E" fields.

Recall: $U_G = mgh$ and equivalently, $U_E = qEd$ (where electric potential difference "V" = Ed).

Another convenient unit of electric energy is the electron-volt (eV). If one elementary charge experiences a potential difference of 1 volt, its energy is equal to 1 eV which equals 1.6×10^{-19} J.

Capacitance:

In the above configuration in which two charged parallel metal plates are separated by a small distance (d), there is a relationship between the potential difference and the total charge. This ratio of the charge on either of the conducting plates and the potential difference between the conductors is called capacitance:

$$C = Q / V$$

Capacitance is measured in units of farads (F) and is a measure of the ability of the plates to store charge. Using algebra, we can derive several simple ways of calculating the capacitance. Since:

$$E = k \cdot Q / d^2 = (1 / 4\pi\varepsilon_0) \cdot Q / d^2 = (1 / \varepsilon_0) \cdot Q / 4\pi d^2 = Q / (\varepsilon_0 \cdot A)$$

The last of the expressions above (where $E = Q / (\varepsilon_o \cdot A)$ is beyond the scope of this course to show the derivation – but nonetheless indicates the electrical field between parallel plates is a function of the area of the plates themselves. Within this uniform electric field between the parallel plates (given by $E = V / d$) and "A" is the area of the rectangular plates and ε_o is a constant called the "permittivity of free-space". This also gives rise to the concept that capacitance is based purely on the geometry of the plates. Therefore, if the potential difference between the two plates is doubled, then the amount of stored charge is also doubled – to keep the capacitance the same. We wouldn't expect the area of the charged plates to change just because we increased the potential difference between the two plates would we? Remember the wave equation? Recall that the speed of a wave in a certain medium is a constant – SO - when the frequency of a wave source in that medium is doubled, the wavelength of the wave would be cut in half so as to maintain the constant speed of the wave in that medium. Here - since the capacitance of a capacitor must remain constant – a larger voltage would allow a larger amount of charge to be stored! The potential difference may then be calculated as:

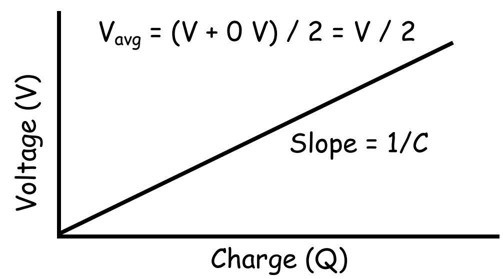
$$V = E \cdot d = Q \cdot d / (\varepsilon_0 \cdot A)$$

And so the Capacitance is therefore:

$$C = Q / V = \varepsilon_0 \cdot A / d$$

To look at the energy stored in a capacitor it is beneficial to use a graphical approach to the charging of the capacitor. A plot of voltage vs. charge for charging a capacitor is a straight line with a slope of $1/C$ - since $V = (1/C) \cdot Q$. The work the battery does gets more difficult as more charges are stored on the plates.

The graph represents the charging of an initially uncharged capacitor ($V_i = 0$ V) to a final voltage (V). The work done is equivalent to transferring the total charge, using an average voltage V_{avg}.



The energy stored in the capacitor (equal to the work done by the battery) is:

$$U_C = W = QV_{avg} = \tfrac{1}{2} QV$$

…and since $Q = CV$, then the energy stored in the capacitor can also be written as:

$$U_C = \tfrac{1}{2} QV = \tfrac{1}{2} Q^2 / C = \tfrac{1}{2} CV^2$$

Millikan's Oil Drop Experiment:

In 1909, American physicist, Robert Millikan, devised a method for measuring the charge of a single electron. Millikan used an "atomizer" (sort-of sprays a very fine mist) to spray drops of oil above a set of two charged plates – like a large capacitor. A hole in the top plate allowed for a few drops of oil to enter the electric field that had been created between the charged plates. Millikan adjusted the charge on the plates until the drops of oil (slightly charged from the friction of the atomizer) were suspended in midair between the plates. The electric field was pushing the drops upward with enough force to exactly overcome the eight of the oil drops. Knowing the potential difference between the plates, Millikan could now measure the magnitude of the electric field that he had created between the plates. To measure the weight (mg) of a single drop, Millikan allowed the drops to fall while he measured the rate at which they fell. Using a complex equation he was able to determine weight. However, when he measured the charges he found there was a wide variety. He added x-rays to ionize the air, removing electrons. When he could narrow down the field he realized that the charges were all multiples of -1.6×10^{-19}. This became the accepted value for the charge of a single electron.

Imagine placing a beaker on a scale to measure its mass. Then place a few marbles in the beaker without counting exactly how many you put in. Record this new mass and then subtract the mass of the beaker. Take the marbles out and add a new (uncounted) quantity. Record this mass – again removing the mass of the beaker. Repeat this procedure for a third time. Once you have three masses recorded see if you can figure out a possible mass for a single marble. Explain why your possible mass may be too big or small. Some important constants are:

Charge On An Electron	Charge On A Proton	Mass Of An Electron	Mass Of A Proton
$q_{e-} = -1.6 \times 10^{-19}$ C	$q_{p+} = 1.6 \times 10^{-19}$ C	$m_{e-} = 9.109 \times 10^{-31}$ kg	$m_{p+} = 1.673 \times 10^{-27}$ kg

Electrodynamics:

1. A proton, starting from rest, falls through a potential difference of 1.0×10^6 V. What is its final kinetic energy? What is its final velocity? *1.6×10^{-13} J 1.38×10^7 m/s*

2. An electron - initially at rest - is accelerated through 1.00 cm by an electric field of 3.00×10^4 V/m. What is its terminal velocity? *1.03×10^7 m/s*

3. An electron is released from rest in a uniform electric field (E = 1.0 N/C). What velocity will it acquire in traveling 1.0 cm? What will then be its kinetic energy? How long a time is required? (Neglect the gravitational force.) *6.0×10^4 m/s 1.6×10^{-21} J 3.4×10^{-7} s*

4. A proton is accelerated from rest for 1.0 nanosecond by an electric field, E = 3.0×10^4 V/m. What is the final velocity? *2.9×10^3 m/s*

5. What potential difference V in an electron gun is required to accelerate an electron that was initially at rest to a final speed of 1.0×10^7 m/s? *285 V*

6. If an electron is projected into an upward electric field with a horizontal velocity v_x, find the equation of its trajectory. (This is analogous to a bullet fired in a gravitational field.)

7. An electron with velocity of 1.0×10^9 cm/s enters a region of 1.0 cm length in which there exists a transverse deflecting field E = 5.7×10^3 V/m. What angle with the x axis does the electron make on leaving the deflecting region? *5.7°*

Millikan Oil Drop & Capacitance:

8. The electric field intensity between two large, charged metal plates is 8000 N/C. The plates are 0.05 m apart. What is the potential difference between them? *400 V*

9. A voltmeter reads 500 V when placed across two charged, parallel plates. The plates are 0.020 m apart. What is the electric field between them? *2.5×10^4 N/C*

10. What potential difference is applied to two metal plates 0.500 m apart if the electric field between them is 2.50×10^3 N/C? *1250 V*

11. What work is done when 5.0 C is raised in potential by 1.5 V? *7.5 J*

12. A drop is falling in a Millikan oil drop apparatus when the electric field is off. What are the forces on the drop, regardless of its acceleration? If it is falling at constant velocity, what can be said about the forces on it?

13. An oil drop weighs 1.9×10^{-15} N. It is suspended in an electric field of 6.0×10^3 N/C. What is the charge on the drop? How many excess electrons does it carry? *3.2×10^{-19} C $2\ e^-$*

14. A positively-charged oil drop weighs 6.4×10^{-13} N. An electric field of 4.0×10^{6} N/C suspends the drop. What is the charge on the drop? How many electrons is the drop missing? *1.6×10^{-19} C* *$1\ e^{-}$*

15. If three more electrons were removed from the drop in the previous problem, what field would be needed to balance the drop? *1.0×10^{6} N/C*

16. A 27 μF capacitor has a potential difference of 25 V across it. What charge is on the capacitor? *0.00068 C*

17. Both a 3.3 μF capacitor and a 6.8 μF capacitor are connected across a 1.5 V potential difference. Which capacitor has a greater charge? What is it? *larger capacitor* *1.0×10^{-5} C*

18. The same two capacitors from the previous problems are each charged to 2.5×10^{-4} C. Across which is the potential difference larger? What is it? *smaller capacitor* *76 V*

19. A 2.2 μF capacitor is first charged so that the potential difference is 6.0 V. How much additional charge is needed to increase the potential difference to 15.0 V? *2.0×10^{-5} C*

20. How many electrons would need to be stored in order to have a full coulomb of charge? How many protons?
6.25×10^{18} electrons

21. What is the numerical value of the permittivity of free-space ε_o – don't forget units!
$8.84 \times 10^{-12}\ C^2/(N \cdot m^2)$

22. During a heart attack, the heart beats in an erratic fashion, called fibrillation. One way to get it back to normal rhythm is to shock it with electrical energy supplied by a cardiac defibrillator. About 1.875×10^{21} eV of energy is required to produce the desired effect. Typically, a defibrillator stores this energy in a capacitor charged by a 5000 V power supply. What capacitance is required? What is the charge on the capacitor's plates?
24 μF *0.120 C*

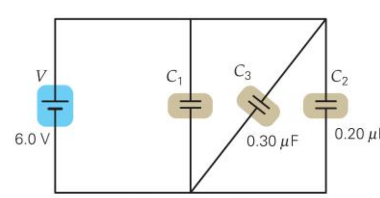
23. A capacitor is first connected to a 6.0 V battery and the disconnected and connected to a 12.0 V battery. How dies its capacitance change? How does the charge of one of its plates change? By how much does the electric field strength between the plates change? *none* *twice* *twice*

24. How much charge flows through a 12 V battery when a 2 μF capacitor is connected across its terminals? *24 μC*

25. What plate separation is required for a parallel-plate capacitor to have a capacitance of 5.0×10^{-9} F if the plate area is 0.40 m^2? *0.71 mm*

26. Capacitors in series have the same? Capacitors in parallel have the same? *charge* *voltage*

27. Using the circuit diagram to the right: What value of C_1 will provide an equivalent capacitance of 1.7 μF? *1.2 μF*



28. If the same three capacitors from the previous problem were connected in series rather than in parallel, what would be the equivalent capacitance?
0.11 μF