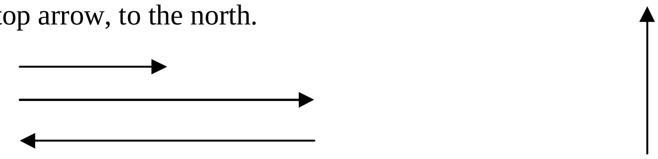
# Vectors and Projectile Motion Notes

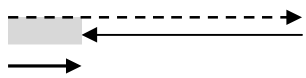
When you use an arrow to represent a vector quantity its length represents the magnitude and the arrow points in the direction of the vector quantity. Look at the arrows below: the top arrow could represent a displacement of 10 meters to the east. Based on that knowledge the middle arrow, which is twice as long as the top arrow, would represent 20 meters to the east. The bottom arrow would also represent 20 meters, but to the west. The arrow pointing to the top of the page would represent 10 meters, since it is the same length as the top arrow, to the north.



The arrow as used here is a vector and can be used to represent any vector quantity. Let's use the arrows above to represent velocities. If the top arrow is 20 mi/h to the south, then the middle arrow would be 40 mi/h to the south. The bottom arrow is 40 mi/h north and the arrow pointing up is 20 mi/h to the east. What the arrow represents depends on how you define (set-up) the original vector representation. Once you have defined the original, all other arrows must be drawn to the same "scale". If one centimeter on your paper represents 10 mi/h, then an arrow representing 80 mi/h must be eight centimeters long. If pointing right represents north, then pointing left must be south.

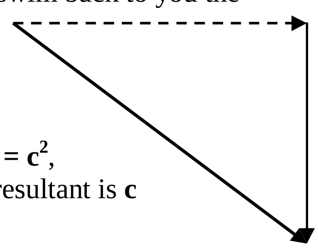
Combining vector quantities graphically is often useful to help "picture" quantities that may seem abstract and not as straight forward. Consider a girl swimming with a velocity of 3 m/s downstream in a river. Her velocity can be represented with a vector three centimeters long. The velocity of the water in the river is 4 m/s.
The river vector is
4 centimeters long.


If we want to know how fast the girl is swimming relative to you watching on the side of the river, we can just combine the vectors. This combination, called the resultant is shown below the combined vectors. The resultant's length is 7 cm, which represents a total observed velocity of 7 m/s. The graphical result is the same as if we simply added the magnitudes of the velocities together. Now, suppose the girl turns around and swims upstream. How fast is the girl swimming relative to you?
Look at the vectors: The shaded area represents the resultant of the combination of the two velocity vectors. The non-shaded area

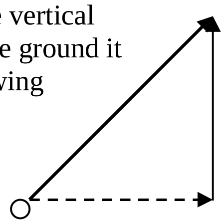


is where the oppositely directed vectors cancel each other out. The resultant vector is 1 cm long, which represents 1 m/s downstream. That's right the girl is swimming but is moving downstream relative to you the observer. The two velocities are in opposite directions and the magnitude of the resultant is just the difference between the two velocities.

The last example involves the situation where the girl turns to swim back to you the observer on the side of the river. The resultant is longer than either of the two vectors, but not as long as when they were in the same direction. The resultant is 5 cm long, which represents 5 m/s. This Value can be calculated using Pythagorean's theorem: $\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$, where the river's velocity is **a**, the girl's velocity is **b**, and the resultant is **c**



A projectile is any object that moves through air or space and is only acted on by gravity and possibly air resistance. In the previous unit the motion of an object in freefall was introduced. An object in freefall is a projectile that only has motion in the vertical dimension. If you were to throw a baseball into the air at an angle with the ground it would have both horizontal and vertical motion. This can be seen by drawing If the velocity vector for the baseball is the hypotenuse of a right triangle, Then the legs of the triangle represent the horizontal and vertical Components of the baseball's motion. The vertical component is Identical to the baseball's motion discussed while introducing the freefall concept. The horizontal component is the same as if the ball were rolled along a smooth table top. Recall from the first marble motion activity that the marble did slow down slightly due to friction between the marble and the table. With a projectile the air will cause some slowing both horizontally and vertically. However, as in the previous unit, we will focus on the scenario where the air resistance has no noticeable effect.



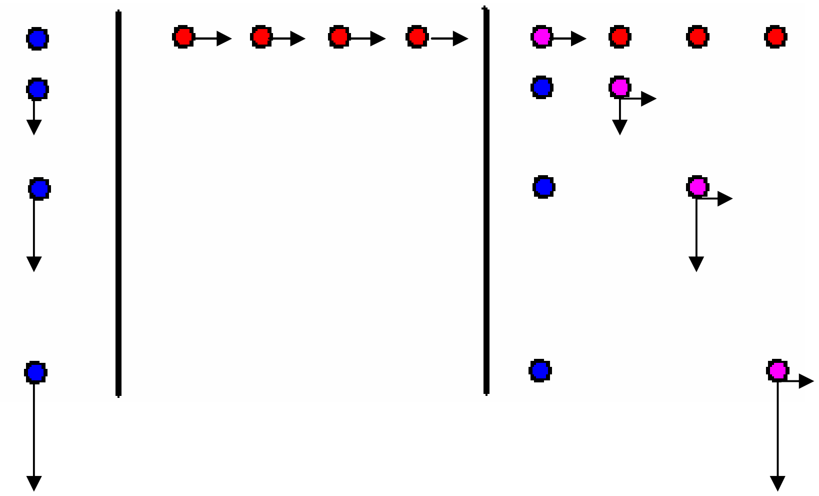
SUPER IMPORTANT STATEMENT ABOUT MOTION IN TWO DIMENSIONS:

The vertical and horizontal components of motion are independent of one another.

What happens in the vertical dimension has no direct effect on what happens in the horizontal dimension.

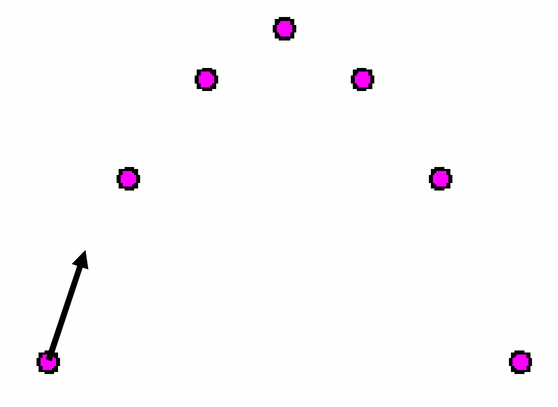
Based on this a projectile's motion can be analyzed as two linear motions, then simply combine the velocity vectors. The vertical component represents the freefall behavior of the object, while the horizontal component represents the horizontal motion, like a ball rolling across a frictionless table.

**Vertical** + **Horizontal** = **Both: Parabolic path**



The first diagram shows the acceleration of freefall, the second shows the constant velocity of frictionless horizontal motion. The final is just like graphing the horizontal and vertical on a graph, the result is the shape of a parabola. The path of every projectile is in the shape of a parabola. The resultant velocity of the projectile at each point of its motion can be found using Pythagorean's theorem. However, the distance the projectile will travel horizontally, the range, depends directly on how fast it is moving horizontally. The time it falls depends directly on its vertical freefall motion. Both motions are happening simultaneously, but independently of each other. An interesting result of the independence of the components is that no matter how fast the projectile travels horizontally it will always strike the ground at the same time based only on its vertical motion. So, a cannonball fired horizontally at 10 m/s would hit the ground at the same time as a cannonball dropped from the same height.

Now for an upwardly launched projectile. Draw the vertical and horizontal velocity vectors at each point of the projectile's path.



As the projectile rises its vertical velocity gets slower at the rate of freefall, but the horizontal velocity remains constant. At the very top of the projectile's flight it has no vertical velocity and all of its velocity is horizontal. On the way down the projectile accelerates vertically based on freefall while still travelling at a constant velocity horizontally. The time the projectile in the air again depends only on the vertical component of its motion. If air resistance were to be considered, both motions would slow down and cause a change in the projectile's path. Picture a strong wind pushing the projectile back towards the starting point.
An interesting symmetry occurs with projectiles when fired at various angles. It turns out that the range of a projectile fired at an angle of 60º with the horizontal will have the same range as an identically fired projectile at an angle of 30º with the horizontal. Notice that the angles add up to 90º, or are complementary angles. This is true for any launch angle and ignoring air resistance.

## Satellite motion

Communication satellites, the international space station, and even the moon are satellites which orbit the earth. These satellites are simply projectiles which are falling around the earth. The moon was not launched from the earth by man, but the other examples were. Isaac Newton proposed a thought experiment using a large cannon on a very high mountain. He ignored air for simplicity sake. If this cannon were to fire a cannonball horizontally at some speed it would follow the rules of motion as described for a projectile and follow a parabolic path to the ground. You could fire the cannonball faster and it would land farther downrange than the first and so on. However, since the earth is basically a big sphere, its surface gradually curves down and away from the cannon. What if the cannonball was fired at such a high speed that by the time it fell the ground itself had curved out of the way? If this were to happen the cannonball would continue to fall until it hit something. It turns out that the earth's curve is approximately such that for every 8 km of horizontal it drops about 5 meters. Recall from our freefall discussion, that an object in freefall falls about 5 meters in one second. So, if the cannonball were fired horizontally at about 8 km/s the ground should curve out of the way by the time the cannonball has travelled the 8 km. This is the basis of satellite motion, simply a projectile falling around another object.